AS and A level Further Mathematics Practice Paper – Vectors – Mark scheme

Question	Scheme	Marks
1(a)	$l_1 : \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}.$ Let $\theta =$acute angle between l_1 and l_2. $\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$ So, $\left\{\overrightarrow{OA}\right\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$	M1A1 **(2)**
1(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$ $\mathbf{k}:\ p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$	M1 M1 A1 **(3)**
1(c)	$\mathbf{d_1} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \ \mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ $\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$ $\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	M1 dM1 A1 **(3)**
1(d)	$\overrightarrow{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}; \ \overrightarrow{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ $\left\|\overrightarrow{AB}\right\| = \sqrt{6^2 + 8^2 + (-10)^2}\ \left\{= 10\sqrt{2}\right\}$ $\qquad \dfrac{d}{10\sqrt{2}} = \sin\theta$ $\left\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\right\} d = 7.456540753... = 7.46$ (3sf)	M1 dM1 A1 **(3)**
		(11 marks)

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Question	Scheme	Marks
2(a)	$\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1 A1 **(2)**
2(b)	$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	M1 A1ft **(2)**
2(c)	$\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	 B1 M1 M1 A1 **(4)**
2(d)	$AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 \quad (= 46)$ $AC = \sqrt{46}$ accept awrt 6.8	M1 A1 **(2)**
		(10 marks)

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Question	Scheme	Marks
3(a)	$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$; $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$ lies on l_1	
	$\{l_1 = l_2 \Rightarrow\}$ $28 - 5\lambda = 3$ $\{\Rightarrow \lambda = 5\}$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ $\{\Rightarrow \mu = -2\}$	B1
	$\{\overrightarrow{OX} =\}$ $\begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$	M1
	So, $X(-1, 3, 9)$	A1 **cao** **(3)**
3(b)	$\mathbf{d_1} = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$	M1
	$\cos\theta = \dfrac{\pm \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}}{\sqrt{(-1)^2 + (-5)^2 + (1)^2}.\sqrt{(3)^2 + (0)^2 + (-4)^2}}$ $\left\{= \dfrac{-7}{\sqrt{27}.\sqrt{25}}\right\}$	dM1
	$\{\theta = 105.6303588... \Rightarrow\}$ $\theta_{\text{Acute}} = 74.36964117... = 74.37$ (2 dp)	A1 **(3)**
3(c)	$\overrightarrow{AX} = "\overrightarrow{OX}" - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix}$ or	
	$A_{\lambda=2}, X_{\lambda=5} \Rightarrow AX = 3\lvert\mathbf{d}_1\rvert$, $\{\lvert\mathbf{d}_1\rvert = \sqrt{27}\}$	
	$AX = \sqrt{(-3)^2 + (-15)^2 + (3)^2}$ or $3\sqrt{27}$ $\{= \sqrt{243}\} = 9\sqrt{3}$	M1 A1 **cao** **(2)**
3(d)	$\dfrac{YA}{"9\sqrt{3}"} = \tan("74.36964...")$	M1
	$YA = 55.71758... = 55.7$ (1 dp)	A1 **(2)**

Question	Scheme	Marks
3(e)	$\left\{A_{\lambda=2},\ X_{\lambda=5} \Rightarrow \text{So } AX = 2AB \Rightarrow \text{So at } B,\ \lambda = 3.5 \text{ or } \lambda = 0.5)\right\}$ $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 3.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}; = \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}; = \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	M1 A1 A1 **(3)**
		(13 marks)

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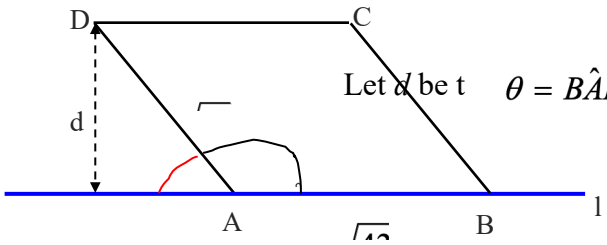
Question	Scheme	Marks
4(a)	**i**: $6-\lambda=-5+2\mu$ **j:** $-3+2\lambda=15-3\mu$ Any two equations leading to $\lambda=3,\ \ \mu=4$ $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ **k:** LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$ (As LHS = RHS, lines intersect)	M1 M1 A1 M1 A1 B1 **(6)**
4(b)	$\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta \quad (\theta\approx 110.92°)$ Acute angle is $69.1°$ awrt 69.1	M1 A1 A1 **(3)**
4(c)	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix} \quad (\Rightarrow B \text{ lies on } l_1)$	B1 **(1)**
4(d)	Let d be shortest distance from B to l_2 $\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$ awrt 7.5 $\dfrac{d}{\sqrt{56}}=\sin\theta$ $d=\sqrt{56}\sin 69.1°\approx 6.99$ awrt 6.99	M1 A1 M1 A1 **(4)**
		(14 marks)

Question	Scheme	Marks
5(a)	**i:** $9+\lambda = 2+2\mu$ **(1)** **j:** $13+4\lambda = -1+\mu$ **(2)** **k:** $-3-2\lambda = 1+\mu$ **(3)** Any two equations. (Allow one slip).	M1
	Eg: **(2) – (3)**: $16+6\lambda = -2$ or **(2) – 4(1)**: $-23 = -9-7\mu$ An attempt to eliminate one of the parameters.	M1
	Leading to $\lambda = -3$ or $\mu = 2$ Either $\lambda = -3$ or $\mu = 2$	A1
	$l_1 : \mathbf{r} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$	M1 A1
		(5)
5(b)	$\mathbf{d_1} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \ \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm\left(\dfrac{2+4-2}{\sqrt{(1)^2+(4)^2+(-2)^2}.\sqrt{(2)^2+(1)^2+(1)^2}}\right)$ Correct **equation**.	A1
	$\cos\theta = \dfrac{4}{\sqrt{21}.\sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1$ (1 dp) awrt 69.1	A1
		(3)
5(c)	$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix}, \ \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix}$	
	$\overrightarrow{AP} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix}$	M1 A1
	$\overrightarrow{AP} \bullet \mathbf{d_1} = 0 \Rightarrow \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \lambda+5+16\lambda-12+4\lambda = 0$	M1
	leading to $\{21\lambda - 7 = 0 \Rightarrow\}$ $\lambda = \dfrac{1}{3}$ $\lambda = \dfrac{1}{3}$	A1
	Position vector $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \dfrac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{28}{3} \\ \frac{43}{3} \\ -\frac{11}{3} \end{pmatrix}$	M1 A1
		(6)
		(14 marks)

Question	Scheme	Marks
6(a)	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\left\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\right\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ $\overrightarrow{AB} = = \pm\left((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})\right); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$	M1 A1 **(2)**
6(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	M1 A1ft **(2)**
6(c)	 Let d be t $\theta = B\hat{A}D$ e from C to l.	
	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$	M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$ Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD} \text{ or } \overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$ Correct followed through expression or **equation**.	A1
	$\cos\theta = \dfrac{-8}{\sqrt{43} . \sqrt{14}} \Rightarrow \theta = 109.029544... = 109 \text{ (nearest °)}$ awrt 109	A1 **cso** **AG**
		(4)

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Question	Scheme	Marks
6(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$	M1 A1 **(2)**
6(e)	Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$ awrt 23.2	M1 dM1 A1 **(3)**
6(f)	$\frac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$ $\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$ awrt 3.54	M1 A1 **(2)**
		(15 marks)

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Question	Scheme	Marks
7(a)	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{AB} = = \pm\left((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k})\right); = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1 A1 **(2)**
7(b)	$\{l_1 : \mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	B1ft **(1)**
7(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$ $\{\cos\theta =\} \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$ Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB} \text{ or } \overrightarrow{BP}\right)$. $\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$	M1 M1 A1 cso **(3)**
7(d)	$\{l_2 : \mathbf{r} =\} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB} \text{ or } \overrightarrow{BP}\right)$. Correct proof	M1 A1 ft **(2)**
7(e)	$\overrightarrow{OC} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ – their $\overrightarrow{AB}$ $\{C(1, 1, 4),\ D(-1, 3, 2)\}$ At least one set of coordinates are correct. Both sets of coordinates are correct.	M1 A1 ft A1 ft **(3)**

Question	Scheme	Marks
7(f)	$\dfrac{h}{\sqrt{(-1)^2+(1)^2+(5)^2}} = \sin\theta$ $\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ $\sqrt{27}\sin(70.5...)$	M1
	$h = \sqrt{27}\sin(70.5...)\left\{= \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{ awrt } 4.9\right\}$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2}\,2\sqrt{6}\left(\sqrt{3}+2\sqrt{3}\right)$ $\dfrac{1}{2}(\text{their } h)(\text{their } AB + \text{their}$	dM1
	$\left\{= \dfrac{1}{2}\,2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$ $9\sqrt{2}$	A1 **cao** **(4)**
		(15 marks)

Question	Scheme	Marks
8(a)	$l: \mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \quad A(3, -2, 6), \quad \overrightarrow{OP} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix}$	
	$\{\overrightarrow{PA}\} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} \qquad \{\overrightarrow{AP}\} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$ Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling.	M1
	$= \begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix} \qquad = \begin{pmatrix} -3-p \\ 2 \\ 2p-6 \end{pmatrix}$ Correct difference.	A1
	$\begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = 6 + 2p - 4 - 6 + 2p = 0$	M1
	$p = 1$	A1 **cso**
		(4)
8(b)	$\lvert AP \rvert = \sqrt{4^2 + (-2)^2 + 4^2}$ or $\lvert AP \rvert = \sqrt{(-4)^2 + 2^2 + (-4)^2}$	M1
	So, PA or $AP = \sqrt{36}$ or 6 **cao**	A1 **cao**
	It follows that, $AB = "6"$ $\{= PA\}$ or $PB = "6\sqrt{2}"$ $\{= \sqrt{2}\,PA\}$	B1 ft
	{Note that $AB = "6" = 2$(the modulus of the direction vector of l)}	
	$\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 7 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ Uses a correct method in order to find both possible sets of coordinates of B.	M1
	$= \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}$ Both coordinates are correct.	A1 **cao**
		(4)
		(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C4 2015	4	6.1, 6.3	Vector equations, scalar products	1.1b, 3.1a
2	C4 Jan 2011	4	6.1, 6.3, 6.4	Vectors	1.1b, 2.1
3	C4 2017	6	6.1, 6.3, 6.4	Vectors	1.1b, 2.1, 3.1a
4	C4 2011	6	6.1, 6.3	Vector equations of lines and planes, Scalar products	1.1b, 2.1, 3.1a
5	C4 Jan 2013	7	6.1, 6.3, 6.4	Vector equations of lines and planes, Scalar products	1.1b, 3.1a
6	C4 Jan 2012	7	6.1, 6.3	Vectors	1.1b, 2.1, 3.1a
7	C4 June 2014	8	6.1, 6.3	Vector equations, scalar products	1.1b, 2.1, 3.1a
8	C4 2013	8	6.1, 6.3, 6.4	Vectors	1.1b, 2.2a, 3.1a